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MUNDELL ONCE AGAIN?:
THE CHOICE OF TARGETS AND INSTRUMENTS IN A SMALL OPEN
ECONOMY WITHOUT CAPITAL MOBILITY*

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Abstract

Using dynamic and stochastic general equilibrium model techniques, I developed a model for a small open economy with a monetary and exchange rate reaction functions with limited capital mobility which excludes the presence of the “impossible trinity problem.” Including main Neo-Keynesian features and a transactional money demand in the framework of a shopping time model, I conclude that co-existence of both rules are possible. Even more and an optimal policy criteria, monetary rule could be focused in controlling liquidity and exchange-rate rule could be related to tradable inflation, in similar way that famous Mundell’s effective market classification principle for instruments allocation.

¿Mundell una vez más?: La elección objetivos e instrumentos en una economía pequeña y abierta sin movilidad de capitales

Resumen

Usando técnicas de modelos de equilibrio general dinámico y estocástico, desarrollo un modelo para una economía pequeña y abierta con funciones de reacción para la cantidad de dinero y el tipo de cambio en un entorno de limitada movilidad de capitales, lo que evita la presencia del “problema de la trinidad imposible.” Incluyendo las principales características de modelos neo-keynesianos y demanda de dinero en el contexto de un modelo de tiempo de compras, concluyo que la coexistencia de ambas reglas es posible. Aún más y con el uso de criterios de política monetaria óptima, la regla monetaria puede concentrarse en el control de la liquidez y la regla cambiaria en el de la inflación transable, en una conclusión similar al “Principio de clasificación efectiva de mercado” de Mundell para la elección de instrumentos.

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I. Introduction

Since 2006, the Bolivian government has employed monetary exchange rate policies: The first one to deal with internal pressures on prices; and the last one intended to cope with the dramatic increase of commodity prices and external inflationary pressures driven by the appreciation of their main partners' currencies.

Although these policies have similar goals, to mitigate inflationary pressures and to promote economic growth during the Great Recession, there was no inconsistency due the absence of the “incompatible or impossible trinity”, i.e. capital mobility and independent monetary and exchange rate policies postulated by the Nobel laureate Robert Mundell in the late of the 60's. In the Bolivian case, international capital mobility is limited, especially in short term flows, due to a low development of financial instruments and markets.

Mundell also proposed a framework to assign policy instruments to targets, focused in monetary and fiscal policies (Mundell, 1962). The structure of this seminal paper (outlined here and in the Appendix 1 following Turnovsky, 1977) assesses that if there are two instruments as the exchange rate and the money supply (e, m), the exchange rate policy could focus only on external inflation and monetary policy on domestic inflation if I assume that exchange rate effect on external inflation (π_F) is bigger than monetary policy in relative terms, that is in charge of controlling domestic inflation (π_H). That is:

$$\frac{\frac{\partial \pi_F}{\partial e}}{\frac{\partial \pi_F}{\partial m}} > \frac{\frac{\partial \pi_H}{\partial e}}{\frac{\partial \pi_H}{\partial m}} \quad (1)$$

In order to assess instruments allocation problem that could be applied to Bolivian economy or other small open economy with limited capital mobility, I build an inter-temporal discrete new Keynesian model to evaluate welfare implications of this policy compared to the use of just one instrument (monetary policy) to control overall inflation. To do so, I employ a Monacelli (1999 and 2004) structure, modified by Parrado and Velasco (2001) and later by Galí and Monacelli (2005), with the introduction of money in the utility function, besides some characteristics of Gertler (2003).

Moreover, I modify these models to not allow capital mobility in two ways. On one hand, there is no arbitrage between domestic and foreign assets; i.e. uncovered interest parity does not hold. On the other hand, households cannot hold foreign assets, which is relevant to the building of the new IS curve.

The paper is organized as follows. After this introduction, I build a basic new Keynesian model applying a Dynamic Stochastic General Equilibrium (DSGE) model specifying households, firms, and central bank behavior. Then I simulate several rules of monetary and exchange rate policies

focusing on welfare implications, including the optimal policy approach. Finally, I compare theoretical findings with the Bolivian experience during the last years.

II. A stylized model for a small open economy without capital mobility

I assume a small open economy without capital mobility, composed of households, firms and central bank (financial public sector).

A) Households:

The representative household seeks to maximize their expected utility based on consumption, money holding and leisure (the opposite of labor), described by the following equation, which has a Constant Relative Risk Aversion (CRRA) form on each of the fundamentals, as it is stated by Gertler *op. cit.*:

$$U = E_t \left(\sum_{i=0}^{\infty} \beta^i \left[\frac{1}{1-\gamma_c} \left(\frac{C_{t+i}}{Q_{t+i}} \right)^{1-\gamma_c} + \frac{a_m}{1-\gamma_m} \left(\frac{M_{t+i}}{Z_{t+i} P_{t+i}} \right)^{1-\gamma_m} - \frac{a_n}{1+\gamma_n} L_{t+i}^{1+\gamma_n} \right] \right) \quad (2)$$

Where Q_t is a consumption preference shock and Z_t is a money preference shock, both at time t . Here, C is a composite consumption bundle defined by the domestic consumption of home (H) and foreign (F) goods, aggregated in a Constant Elasticity Substitution (CES) way, such that:

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (3)$$

Where α ($1-\alpha$) is the share of domestic (foreign) produced goods consumed domestically and $C_{H,t}$ and $C_{F,t}$ are consumption of domestic and foreign goods consumed at home. Both of them are a composite good of many varieties with a CES specification:

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (4)$$

$$C_{F,t} \equiv \left(\int_0^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (5)$$

That is, there is a continuum of goods varieties within the range $j \in [0, 1]$. In both, ε is the elasticity of substitution between these varieties.

This household will face this budget constraint:

$$\int_0^1 (P_{H,t}(j) \times C_{H,t}(j) + P_{F,t}(j) \times C_{F,t}(j)) dj + M_t + \frac{1}{1+i_t} B_t \quad (6)$$

$$\leq W_t L_t + \Pi_t + M_{t-1} + B_{t-1} + TR_t$$

B_t denotes the purchase of home bonds at time t . The economy is closed in terms of inflows and/or outflows of capital from/to the rest of the world, meaning that domestic households will only be able to purchase domestic bonds. M_t is the nominal value of money used by this household to make its transactions, W_t is the nominal wage paid to this household and Π_t is the profit of domestic firm owned by this representative household.

Therefore the expected value of bonds purchased at t depends solely on the domestic interest rate, which is not influenced by the world interest rate, due the absence of capital mobility abroad.

The consumer demand for each variety j produced at home and foreign, as described by Dixit-Stiglitz (1977) is obtained through a minimization process, with the following results:

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} C_{H,t} \quad (7)$$

$$C_{F,t}(j) = \left[\frac{P_{F,t}(j)}{P_{F,t}} \right]^{-\varepsilon} C_{F,t} \quad (8)$$

Where $P_{H,t}$ and $P_{F,t}$ are price indexes of domestic and foreign produced goods in terms of domestic currency. These are written as:

$$P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \quad (9)$$

$$P_{F,t} \equiv \left[\int_0^1 P_{F,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \quad (10)$$

As usual, from these I obtain the overall Consumer Price Index (P) as a composite of domestic price index ($P_{H,t}$) and foreign price index ($P_{F,t}$). That it is:

$$P \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (11)$$

Given previous definitions, I can replace $\int_0^1 [P_{H,t}(j)C_{H,t}(j) + P_{F,t}(j)C_{F,t}(j)]$ in the budget constraint by $P_t C_t$ so I have a reduced form of the budget constraint which states that consumption in each period t has to be lower or equal than the sum of labor income, profit, government transfer minus the accumulation of money and bond holdings:

$$P_t C_t \leq W_t N_t + \Pi_t + TR_t - (M_t - M_{t-1}) - \left(\frac{B_t}{1+i_t} - B_{t-1} \right) \quad (12)$$

Then I am able to state the inter-temporal maximization problem restrained by the budget constraint:

$$E_t \left(\sum_{i=0}^{\infty} \beta^i \left[\frac{1}{1-\gamma_c} \left(\frac{C_{t+i}}{Q_{t+i}} \right)^{1-\gamma_c} + \frac{a_m}{1-\gamma_m} \left(\frac{M_{t+i}}{Z_{t+i} P_{t+i}} \right)^{1-\gamma_m} - \frac{a_n}{1+\gamma_n} L_{t+i}^{1+\gamma_n} \right] + \lambda_t \left(P_t C_t + M_t + \frac{1}{1+i_t} B_t - W_t L_t + \Pi_t + M_{t-1} + B_t + TR_t \right) \right) \quad (13)$$

The First Order Conditions (FOC) are:

$$1 = E_t \left\{ (1+i_t) \left(\frac{P_t}{P_{t+1}} \right) \beta \left(\frac{C_t}{C_{t+1}} \times \frac{Q_{t+1}}{Q_t} \right)^{-\gamma_c} \right\} = E_t \left\{ \beta (1+r_t) \left(\frac{C_t}{C_{t+1}} \times \frac{Q_{t+1}}{Q_t} \right)^{-\gamma_c} \right\} \quad (14)$$

$$\left(\frac{C_t}{Q_t} \right)^{-\gamma_c} \left(\frac{i_t}{1+i_t} \right) = a_m \left(\frac{M_t}{Z_t P_t} \right)^{-\gamma_m} \quad (15)$$

$$\frac{W_t}{P_t} = \frac{a_n L_t^{\gamma_n}}{C_t^{-\gamma_c}} \quad (16)$$

The first one is the Euler equation, where r_t is the real interest rate defined as $(1+i_t)/(1+\pi_{t+1})$, and the overall inflation rate is $\pi_t = P_t/P_{t-1} - 1$. The second one is the money demand equation and the last equation is the labor supply.

Finally I log-linearize these equations to use them later, with the usual techniques applied to DSGE models, where q is a preference shock:¹

$$\tilde{c}_t = -\sigma [\tilde{i}_t - E_t \tilde{\pi}_{t+1}] + E_t \tilde{c}_{t+1} + \chi_t \quad / \sigma = 1/\gamma_c \wedge \chi_t = (1+\rho_q) q_t \quad (17)$$

$$\tilde{m}_t - \tilde{p}_t = \alpha_y \tilde{c}_t - \alpha_m \tilde{i}_t + \iota_t \quad / \alpha_y = \gamma_c / \gamma_m, \alpha_m = \gamma_m / [(1+\bar{i}) \times \bar{i}], \iota_t = \alpha_y q_t + \gamma_m z_t \quad (18)$$

$$\tilde{w}_t - \tilde{p}_t = \gamma_c \tilde{c}_t + \gamma_n \tilde{l}_t \quad (19)$$

¹ In the case of a variable x , \bar{x} is the steady state equilibrium value and \tilde{x} is the log deviation from it.

It could be proved that the assignment of consumption between home and foreign goods is given by:

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad (20)$$

$$C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \quad (21)$$

These equations will be complemented with overall price index in the log-linear form:

$$\tilde{p}_t = (1-\alpha) \tilde{p}_{H,t} + \alpha \tilde{p}_{F,t} \quad (22)$$

And one expression for the inflation rate:

$$\tilde{\pi}_t = \tilde{p}_t - \tilde{p}_{t-1} = (1-\alpha) \tilde{\pi}_{H,t} + \alpha \tilde{\pi}_{F,t} \quad (23)$$

B) Firms

As there is a continuum of varieties, I will assume each domestic firm only produces one variety. The production function of variety j is defined as: $Y_{H,t} = A_t L_t(j)$ where A_t is productivity of labor and it follows an AR(1) process of the following form: $a_t = \ln(A_t) = \rho_A a_{t-1} + v_t$.

Given the assumption of monopolistic competition, each firm has the following demand function:

$$Y_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \quad (24)$$

Using the stickiness framework provided by Calvo (1983) and in line with Parrado and Velasco *op. cit.* approach, the problem of a representative firm is stated as:

$$\text{Max } E_{t-1} \sum_{k=0}^{\infty} (\theta\beta)^k \Lambda_{t+j} \left\{ \frac{P_{H,t}(j)}{P_{H,t+k}} \left(\frac{P_{H,t}(j)}{P_{H,t+k}} \right)^{-\varepsilon} C_{H,t+k} - \frac{W_{t+k}}{P_{H,t+j}} \frac{V \left(\frac{P_{H,t}(j)}{P_{H,t+k}} \right)^{-\varepsilon} C_{H,t+k}}{A_t} \right\} \quad (25)$$

Where θ is the (exogenous) probability that the firm will maintain the price, which follows a Bernoulli distribution. The term Λ_t joined to β is equal to the appropriate discount factor for this consumer-producer problem. Then the FOC will be:

$$E_{t-1} \sum_{k=0}^{\infty} (\theta\beta)^k \Lambda_{t+j} \left\{ \begin{array}{l} \frac{1}{P_{H,t+k}} \left(\frac{P_{H,t}(j)}{P_{H,t+k}} \right)^{-\varepsilon} C_{H,t+k} - \varepsilon \frac{P_{H,t}}{P_{H,t+k}} \left(\frac{P_{H,t}(j)}{P_{H,t+k}} \right)^{-\varepsilon-1} \frac{1}{P_{H,t+k}} C_{H,t+k} \\ + \frac{W_{t+k}}{P_{H,t+j}} \frac{V' \left(\frac{P_{H,t}(j)}{P_{H,t+k}} C_{H,t+k} \right)^{-\varepsilon}}{A_t} \left(\frac{P_{H,t}(j)}{P_{H,t+k}} \right)^{-\varepsilon-1} \frac{1}{P_{H,t+k}} \end{array} \right\} = 0 \quad (26)$$

After some algebraic manipulation, it can be proved that the last expression could be simplified in this expression:

$$\bar{P}_{H,t}(j) = (1 + \mu) E_t \sum_{k=0}^{\infty} \omega_{t,t+k} MC_{t+k} \quad (27)$$

Where $\bar{P}(j)$ is the optimal price for variety j , $\mu = (\varepsilon/\varepsilon - 1) - 1$, is the markup related to the monopolistic power of the firm and MC_t is the (nominal) marginal cost at time t . In the previous expression, ω_t is a weight which is function of the parameters in the maximization problem.

In log, this optimal price becomes:

$$\bar{p}_{H,t} = (1 - \theta\beta) E_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k mc_{t+k} \right\} \quad (28)$$

This rational expectation equation could be written in terms of (log-linear) variation as

$$\tilde{\pi}_{H,t} = \delta mc_t + \beta E_t \tilde{\pi}_{H,t+1} \quad / \quad \delta = \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \quad (29)$$

As I assumed a linear production function related just to the labor, then (log-linear) marginal cost is:

$$mc_t = \tilde{w}_t - \tilde{p}_{H,t} - \tilde{a}_t = (\tilde{w}_t - \tilde{p}_t) + (\tilde{p}_t - \tilde{p}_{H,t}) - \tilde{a}_t = (\tilde{w}_t - \tilde{p}_t) + \alpha(\tilde{p}_{F,t} - \tilde{p}_{H,t}) - \tilde{a}_t \quad (30)$$

Defining (the log of) terms of trade s_t as $p_{F,t} - p_{H,t} = (e_t + p_t^* - p_{H,t})$, and substituting the real wage from expression (19), I can transform this into:

$$mc_t = \gamma_c \tilde{c}_t + \gamma_n \tilde{l}_t + \alpha \tilde{s}_t - \tilde{a}_t = \gamma_c \tilde{c}_t + \gamma_n \tilde{y}_t + \alpha \tilde{s}_t - (1 + \gamma_n) \tilde{a}_t \quad (31)$$

Then I define output as equal to consumption (c_t) plus net exports (nx_t). As it is shown in Galí *op. cit.*, the last one is a function of the terms of trade. Then output can be written as:

$$\tilde{y}_t = \frac{\bar{C}}{\bar{Y}} \tilde{c}_t + \frac{\overline{NX}}{\bar{Y}} nx_t = \tilde{c} \times \tilde{c}_t + nx \times nx_t = \tilde{c} \times \tilde{c}_t + nx \times \alpha \left(\frac{\omega}{\gamma_c} - 1 \right) \tilde{s}_t \quad (32)$$

Where $\omega = \gamma_c \varepsilon + (1 - \alpha)(\gamma_c \eta - 1)$. In order to obtain the marginal cost as a function of output gap and terms of trade, I replace consumption from (32) into (31) with the following result:

$$mc_t = \left(\frac{\gamma_c}{\tilde{c}} + \gamma_n \right) \tilde{y}_t + \left[\frac{\alpha}{\tilde{c}} (\tilde{c} - nx(\omega - \gamma_c)) \right] \tilde{s}_t - (1 + \gamma_n) \tilde{a}_t \quad (33)$$

Then, the new Keynesian Phillips curve for domestic prices will be:

$$\tilde{\pi}_{H,t} = \beta E_t \tilde{\pi}_{H,t+1} + \kappa \tilde{y}_t + \lambda (\tilde{e}_t + \tilde{p}_t^* - \tilde{p}_{H,t}) - \zeta \tilde{a}_t \quad (34)$$

With $\kappa = \delta \left(\frac{\gamma_c}{\tilde{c}} + \gamma_n \right)$, $\lambda = \delta \frac{\alpha}{\tilde{c}} (\tilde{c} - nx(\omega - \gamma_c))$ and $\zeta = 1 + \gamma_n$

This NKPC depends solely on the expected domestic inflation, the output gap, terms of trade disaggregated in its components (exchange rate combined with foreign and domestic) prices and productivity shock.

C) Monetary and exchange rate policy

I will assume that the financial public sector (i.e. government) is financed by money creation (through the high-powered money) and they make lump sum transfers to the households, such that:

$$\frac{H_t - H_{t-1}}{P_t} = TR_t \quad (35)$$

In order to find the equilibrium in the money market, I will take the money supply expression found in Appendix B, which is the following:

$$\tilde{m}_t = \tilde{h}_t + \frac{\phi}{cr + rr + \phi \bar{i}} \tilde{i}_t \quad (36)$$

Finally, I need a rule for the monetary policy. One option could be the usual Taylor Rule (**TR**) for the interest rate:

$$\tilde{i}_t = \zeta \tilde{i}_t + (1 - \zeta) [\chi_y^m \tilde{y}_t + \chi_\pi^m \tilde{\pi}_t] = \rho_i \tilde{i}_{t-1} + \rho_y \tilde{y}_t + \rho_\pi \tilde{\pi}_t \quad (37)$$

Other option is a Friedman rule (**FR**) for the growth of high-powered money, with a constant rate of growth (g):

$$\tilde{h}_t = \tilde{h}_{t-1} + g \quad (38)$$

Also I could use a rule for the high-powered money (**MR**), but variable in function to deviations of inflation and output gap. To build this, in the Appendix B I derive a rule consistent with the interest rate rule, which is:

$$\begin{aligned} \tilde{h}_t &= \tilde{p}_t - \underbrace{\left[(\alpha_m + \xi) \rho_\pi^m \right]}_{a_1} \tilde{\pi}_t + \underbrace{(\alpha_y - (\alpha_m + \xi) \rho_y^m)}_{a_2} \tilde{y}_t - \underbrace{(\alpha_m + \xi) \rho_i}_{a_3} \tilde{i}_{t-1} + \psi_t \\ &= \tilde{p}_t - a_1 \tilde{\pi}_t + a_2 \tilde{y}_t - a_3 \tilde{i}_{t-1} + \psi_t \end{aligned} \quad (39)$$

At this point, I have to mention that the use of high-powered money control instead of an interest rate rule would rely in the fact that money demand volatility is assumed to be high, as it be seen in the Bolivian case due dollarization. A basic analysis about the preference between interest and money rule is found in the Appendix C.

Finally, based on the MR rule, I can postulate an optimal rule where high-powered money is based on control of domestic inflation (**HMR**) joint an exchange rate rule of foreign inflation that it will be stated later:

$$\tilde{h}_t = \tilde{p}_t - b_1 \tilde{\pi}_{H,t} + b_2 \tilde{y}_t - b_3 \tilde{i}_{t-1} + \tau_t \quad (40)$$

In the case of exchange rate, I will assume three kinds of rules: i) fixed exchange rate (**FIX**) or $\tilde{e}_t = 0$; ii) a constant competitiveness rule (**CCR**) or $\tilde{e}_t = \tilde{p}_t - \tilde{p}_t^*$; and iii) an exchange rate rule based on foreign inflation and output gap consideration (**ERR**) or $\tilde{e}_t = \rho_e \tilde{e}_{t-1} + (1 - \rho_e) (\rho_{\pi_F}^e \tilde{\pi}_{F,t} + \rho_y^e \tilde{y}_t)$.

To finish policies specification, I will derive optimal policies with two variants: a) both monetary and exchange rate policies focused on inflation and output gap (**OP**); and, b) optimal monetary and exchange rate policies when they are focused in the target where they have more influence, in the aim of Mundell's work (**MOP**).

D) Model specification and parameterization

The basic equations of this DSGE are summarized as follows:

<i>Euler equation:</i>	$\tilde{c}_t = -\sigma \left[\tilde{i}_t - E_t \tilde{\pi}_{t+1} \right] + E_t \tilde{c}_{t+1} + \chi_t$
<i>Money demand (MD):</i>	$\tilde{m}_t = \tilde{p}_t + \alpha_y \tilde{c}_t - \alpha_m \tilde{i}_t + \iota_t$
<i>(Interest rate consistent with MD)</i>	$\tilde{i}_t = \frac{1}{\alpha_m} \left(\tilde{p}_t + \alpha_y \tilde{c}_t + \tilde{i}_t - \tilde{m}_t \right)$
<i>Labor supply:</i>	$\tilde{w}_t = \tilde{p}_t + \gamma_c \tilde{c}_t + \gamma_n \tilde{l}_t$

Phillips Curve:
$$\tilde{\pi}_{H,t} = \beta E_t \tilde{\pi}_{H,t+1} + \kappa \tilde{y}_t + \lambda (\tilde{e}_t + \tilde{p}_t^* - \tilde{p}_{H,t}) - \zeta \tilde{a}_t$$

Net exports:
$$nx_t = \alpha \left(\frac{\omega}{\gamma_c} - 1 \right) \tilde{s}_t$$

Production function:
$$\tilde{y}_t = \tilde{l}_t + a_t$$

Productivity shock:
$$a_t = \rho_A a_{t-1} + v_t a$$

Terms of trade:
$$\tilde{s}_t = \tilde{p}_{F,t} - \tilde{p}_{H,t} = (\tilde{e}_t + \tilde{p}_t^* - \tilde{p}_{H,t})$$

Foreign prices definition :
$$\tilde{p}_{F,t} = \tilde{e}_t + \tilde{p}_t^*$$

Foreign price s shock
$$\tilde{p}_t^* = \rho^* \tilde{p}_{t-1}^* + u_t$$

In order to solve the model, I first need to draw values for each of the coefficients that describe it. To do so, I will draw some of the parameters from studies related to the Bolivian case if it is possible. In other cases, I will use international parameters.

Variable	Value	Parrado and Velasco (PV)	Gali	Valdivia	Source used in this paper
β	0.99	$\beta = 0.99$	$\beta = 0.99$	$\beta = 0.89$	PV
α	0.5	$\gamma = 0.29$	$\alpha = 0.4$	-	Share in Bolivian CPI
θ	0.5	$\alpha = 0.75$	$\theta = 0.75$	0.52	Valdivia
ε	5	$\theta = 4.33$	$\varepsilon = 6$	-	Assumption
γ_c	1	$\sigma = 1$	$\sigma = 1$	$\sigma = 2$	PV and Velásquez and Gallopo (2012)
γ_m	1	-	-	-	Implicit from own money demand estimates and γ_c
γ_n	1.7	-	$\varphi = 3$	$\varphi = 1.7$	Valdivia
η	2	$\eta = 1.5$	$\eta = 1$	$\sigma_s = 2$	Valdivia
ρ^*	0.74	-	-	-	First AR(1) of cyclical component
ρ_a	0.65	$\sigma_z = 0.8$	-	-	First AR(1) of GDP cycle
\bar{i}	0.11	-	-	-	1990-2011 average
\hat{c}	0.94	-	-	-	2004-2011 average
nx	0.06	-	-	-	2004-2011 average
ρ_π^m	1.5	$\chi_\pi = 1.5$	$\phi_\pi = 1.5$	-	PV
ρ_y^m	0.5	$\chi_x = 0.5$ or 0	-	-	PV
ϕ	0.04	-	-	-	Cernadas (2012)
rr	0.15	-	-	-	May 2012 figure
cr	0.39	-	-	-	May 2012 figure
ρ_q	0.3	-	-	-	First AR(1) of cycle
σ_i	0.3	-	-	-	Implicit from own money demand estimates*

*Including de-dollarization this parameter goes down to 0.06

III. Alternative monetary and exchange rate policies welfare implications

To evaluate alternative monetary and exchange rate policies, as Parrado and Velasco *op. cit.* I will use this welfare criterion based on variances of consumption, inflation and terms of trade:

$$E[L] = \Gamma_{\pi} \text{Var}(\tilde{\pi}_{H,t}) + \Gamma_{\tilde{y}} \text{Var}(\tilde{y}_t) \quad (41)$$

I will depart with $\Gamma_{\pi} = \Gamma_{\tilde{y}} = 1$ to run with 100,000 simulations of these policies, whose results are reported in the next table:²

	FIX	CCR	ERR*
FR	0.0823	0.0620	0.0880
MR	0.0806	0.0445	0.0591
HMR	0.0593	0.0445	0.0591
TR	0.0425	0.0258	0.0419

* I use estimates from Mendieta and Palmero (2011): $\tilde{e}_t = 0.75\tilde{e}_{t-1} + 0.25(-0.1\tilde{\pi}_{F,t} - 0.2\tilde{y}_t)$

Other choice is to use an optimal policies approach. With a quadratic loss function, weighted with a matrix \mathbf{W} with endogenous variables vector \mathbf{y}_t , including a vector of instruments with parameters Ξ , I can state the problem as:

$$\max_{\Xi} E(\mathbf{y}_t' \mathbf{W} \mathbf{y}_t) \quad s.t. \quad \Omega_1 E_t \mathbf{y}_{t+1} + \Omega_2 \mathbf{y}_t + \Omega_3 \mathbf{y}_{t-1} + \Omega_4 \mathbf{e}_t = \mathbf{0} \quad (42)$$

Without restraints to the movement of instruments, simulation shows a faster and higher response to deviations of inflation and output, with a loss near to 0, smaller than the previous shown. Parameters seem implausible to apply in the Bolivian economy.

Then, I specify an alternative loss function, where I include losses related to the movement of instruments:

$$E[L] = \Gamma_{\pi} \text{Var}(\tilde{\pi}_{H,t}) + \Gamma_{\tilde{y}} \text{Var}(\tilde{y}_t) + \left\langle \Gamma_e \text{Var}(\Delta \tilde{e}_t) + \begin{Bmatrix} \Gamma_{\tilde{i}} \text{Var}(\Delta \tilde{i}_t) \\ \Gamma_{\tilde{h}} \text{Var}(\Delta \tilde{h}_t) \end{Bmatrix} \right\rangle \quad (43)$$

This procedure results in an initial loss bigger than the previous ones, at least 0.1. This loss comes from the inertia involved in the dynamics of instruments. It is interesting to note the implicit parameters of reaction functions:³

$$\tilde{i}_t = 0.56 \underset{(0.75)}{\times} \tilde{i}_t + 0.44 \left[\underset{(0.5)}{0.65} \times \tilde{y}_t + \underset{(1.5)}{0.28} \times \tilde{\pi}_{H,t} \right] \quad (44)$$

² It must be noted that this table shows 100*E[L].

³ Inside parenthesis are shown parameters used previously. An in order to compare results, I write the equivalent Taylor rule fully consistent with high-powered money rule.

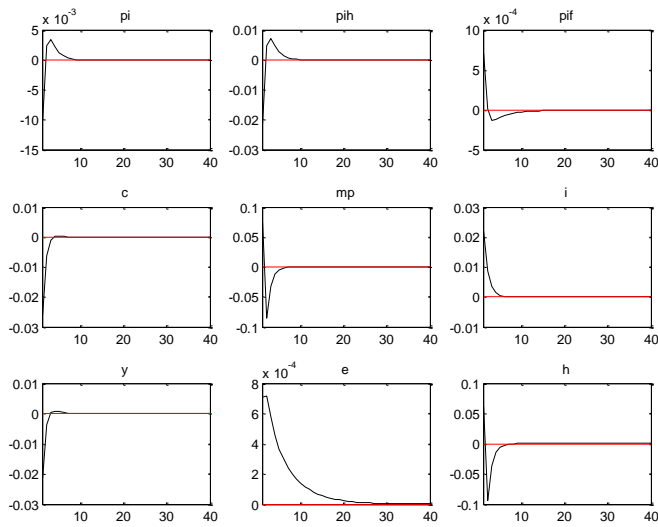
$$\tilde{e}_t = \underset{(0.95)}{0.82} \times \tilde{e}_{t-1} + 0.18 \times \begin{pmatrix} -0.2 \times \tilde{y}_t - 0.2 \times \tilde{\pi}_{F,t} \\ (-0.2) \quad (-0.1) \end{pmatrix} \quad (45)$$

There are some insights about these reaction functions. First, both require less inertia. Second, the weight of domestic inflation in the monetary rule is lower, because it is helped with the exchange rate, which also contains the output gap. Finally, exchange rate policy weights are similar to those used by Bolivian authorities since 2006.

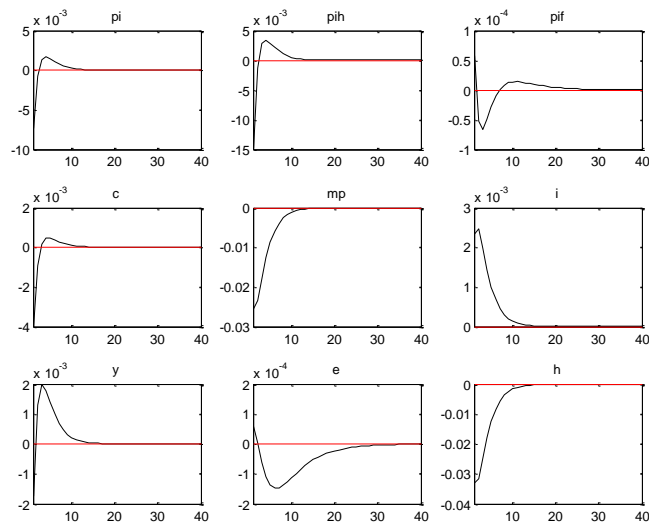
The impulse response functions related to these rules are depicted:

Figure 1: Impulse responses to shocks
(Deviations from the equilibrium)

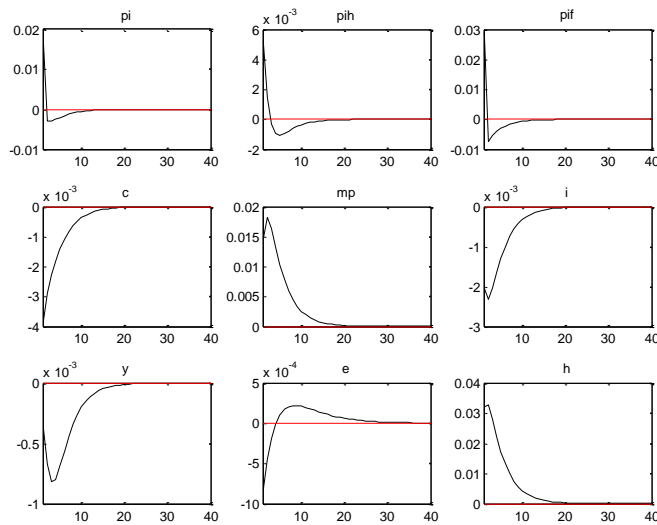
a) Foreign inflation shock



b) Consumption (domestic demand) shock



c) Productivity shock



Source: Own estimates

IV. Inflation and Bolivian Economic Policies⁴

This section describes briefly monetary and exchange rate policies in Bolivia during the last years to compare them later with theoretical findings.

A) Monetary Policy

In the period from 2006 to the present, Bolivian Central Bank's monetary policy has aimed to foster price stability in order to contribute to the social and economic development, according which is stated in the Constitution (2009) and in the Central Bank's Law (1995). It has focused their efforts to control inflation using open market operations as well as foreign exchange rate control.

During this period, there have been two inflationary upsurges in 2007/2008 and 2010/2011. Both peaks were caused by the prompt increase of international food and energy prices that translated into an increase in the domestic price level. Likewise, the volatility of inflation rose in a moderate manner and was, in great part, caused by transitory shocks that were amended right after. Moreover, this rise in the volatility of inflation was not specific to Bolivia; the same effect was seen in neighboring South American countries.

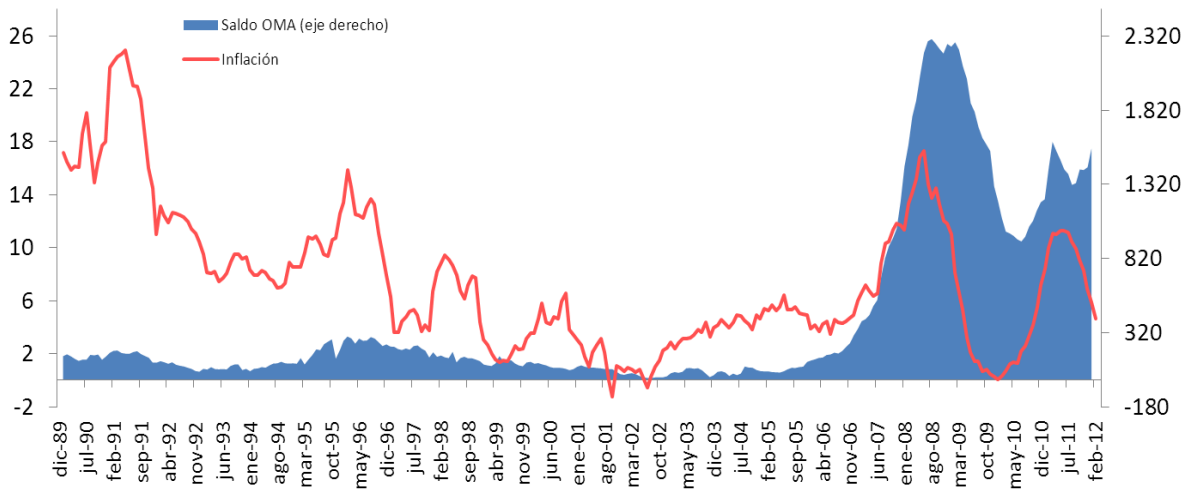
There have also been some innovations and changes in the monetary policy regime in recent years. One of the most influential changes was the de-dollarization process which Bolivia underwent in order to deepen the use of its national currency since 2006. This process has helped improve the transmission mechanisms as well as the efficiency of monetary policy in the Bolivian economy.

⁴ This section is just focused in Central Bank policies. There were also other policies, especially from the Government.

Another factor that has improved the margin of monetary policy in Bolivia has been the deepening and integration of the national financial market.

With this increase in efficiency, the Central Bank has used monetary policy in a more active way. Open market operations since 2006 have increased in a more significant way to control inflation. Moreover, monetary policy did not only react to inflationary pressures, but it has also been used to provide a small boost during periods of economic slowdown like in mid-2007 and mid-2009 (Figure 2).

Figure 2: Inflation and OMO in Bolivia
(Millions of Bolivianos and percentage)



Source: Central Bank of Bolivia and National Institute of Statistics

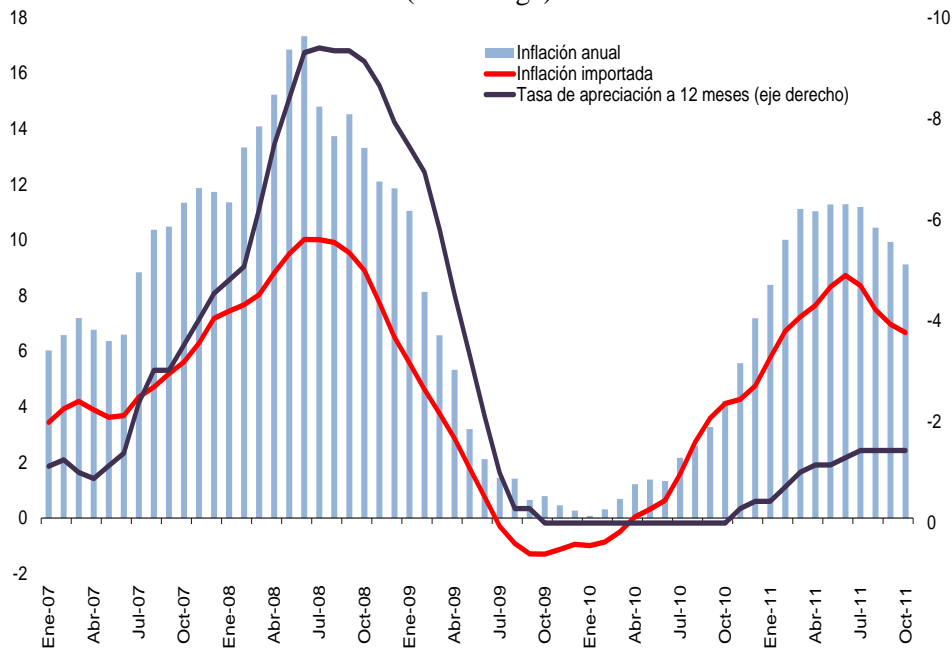
B) Exchange Rate Policy

The evolution of the foreign exchange policy in recent periods can be divided in two periods. Before the year 2006, it was characterized by continuous and gradual depreciations of the national currency in order to keep a stable real exchange rate.

Starting from 2006, this was reversed to a gradual appreciation of the national currency in order to contain external inflationary pressures. The end of 2005 was marked by high increases of the price levels due to these increases in international prices. Therefore, starting from mid-2005, the Boliviano started to appreciate to contribute to mitigate external inflation.

This appreciation period stopped in 2009, where it was fixed in order to cope with changing expectations due to deteriorating international financial market strength. The appreciation of the Boliviano was restarted in 2011 to keep coping with external inflationary pressures (Figure 3). This has also brought a great accumulation of international reserves.

Figure 3: Domestic and imported inflation and Boliviano's appreciation
(Percentage)



Source: Central Bank of Bolivia and National Institute of Statistics

V. Concluding remark

In most of countries central bank policies have been focused in monetary one, letting the exchange rate policy aside because capital mobility do not allowed this to have independency. But in some cases, as the Bolivian one, there is low or limited capital mobility.

Then the paper has focused in the determination of a set of these policies. With the use of a New-Keynesian DSGE framework, assessed these policies from a welfare perspective. Given the characteristics of Bolivian economy, it can be stated that the change from a policy close to Friedman's famous rule with a competitiveness exchange rate policy (FR+CCR) to a home domestic inflation oriented monetary policy plus an exchange rate rule (HMR+ERR) was beneficial. Other theoretical finding is that in order to improve central bank policies, less inertia would be desirable and a gradual movement to an interest rate rule (TR+CCR), according to the deepening of financial and monetary markets and the strengthening of the de-dollarization process.

During the transition, monetary policy could be addressed or compared with explicit rules. As the paper do not included dollarization and other features of Bolivian, obviously results must be taken with caution.

Overall, the Bolivian experience during these last years has shown that it is possible (and even advisable) for a small open country with very limited capital mobility to have an independent monetary policy to cope with internal inflation and at the same time apply foreign exchange rate interventions to avoid external inflationary pressures.

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Appendix A: An application of the effective market classification principle⁵

In this section, I will apply Mundell's (1962) general principle of effective market classification to demonstrate that if condition (46) holds and if monetary policy and exchange rate policy are two available (and independent) instruments, then exchange rate policy could focus only on external inflation and monetary policy on domestic inflation.

Let us suppose that the structure that relates foreign and domestic inflation (π_F and π_H respectively) with monetary (m) and foreign exchange rate policy (e) is as follows:

$$\begin{aligned}\pi_H &= F(m, e) \\ \pi_F &= G(m, e)\end{aligned}\tag{47}$$

Let us assume that the monetary authority sets inflationary targets such that:

$$\begin{aligned}\bar{\pi}_H &= F(\bar{m}, \bar{e}) \\ \bar{\pi}_F &= G(\bar{m}, \bar{e})\end{aligned}\tag{48}$$

The question to address is how instruments could be assigned to these targets. For this I will define the change in money supply and foreign exchange rate as $\dot{m} = \frac{dm}{dt}$ and $\dot{e} = \frac{de}{dt}$.

I suppose that monetary authority reacts to deviations from their objectives, such that:

$$\begin{aligned}\dot{m} &= \frac{dm}{dt} = \alpha_{11}(\pi_H - \bar{\pi}_H) + \alpha_{12}(\pi_F - \bar{\pi}_F) \\ \dot{e} &= \frac{de}{dt} = \alpha_{21}(\pi_H - \bar{\pi}_H) + \alpha_{22}(\pi_F - \bar{\pi}_F)\end{aligned}\tag{49}$$

Replacing this expression with (47) I get:

$$\begin{aligned}\dot{m} &= \alpha_{11}(F(m, e) - \bar{\pi}_H) + \alpha_{12}(G(m, e) - \bar{\pi}_F) \\ \dot{e} &= \alpha_{21}(F(m, e) - \bar{\pi}_H) + \alpha_{22}(G(m, e) - \bar{\pi}_F)\end{aligned}\tag{50}$$

Now I will use a Taylor expansion with two variables to approximate $F(m, e)$ and $G(m, e)$.

$$\begin{aligned}F(m, e) &= F(\bar{m}, \bar{e}) + F_m(m - \bar{m}) + F_e(e - \bar{e}) \\ &= \bar{\pi} + F_m(m - \bar{m}) + F_e(e - \bar{e})\end{aligned}\tag{51}$$

⁵ This Appendix follows closely Turnovsky *op. cit.* analysis.

$$\begin{aligned}
G(m, e) &= G(\bar{m}, \bar{e}) + G_m(m - \bar{m}) + G_e(e - \bar{e}) \\
&= \bar{\pi} + G_m(m - \bar{m}) + G_e(e - \bar{e})
\end{aligned} \tag{52}$$

Where $F_m = \left. \frac{dF}{dm} \right|_{m=\bar{m}}$ and $F_e = \left. \frac{dF}{de} \right|_{e=\bar{e}}$.

Substituting (51) and (52) into (50) I get:

$$\begin{aligned}
\dot{m} &= \alpha_{11}(F_m(m - \bar{m}) + F_e(e - \bar{e})) + \alpha_{12}(G_m(m - \bar{m}) + G_e(e - \bar{e})) \\
\dot{e} &= \alpha_{21}(F_m(m - \bar{m}) + F_e(e - \bar{e})) + \alpha_{22}(G_m(m - \bar{m}) + G_e(e - \bar{e}))
\end{aligned} \tag{53}$$

If I define $\tilde{m} = m - \bar{m}$ and $\tilde{e} = e - \bar{e}$ as the deviations from their equilibrium values, I can prove that $\dot{\tilde{m}} = \dot{m}$ and $\dot{\tilde{e}} = \dot{e}$. If I also group terms, the system will be a differential equations system such that:

$$\begin{aligned}
\dot{\tilde{m}} &= (\alpha_{11}F_m + \alpha_{12}G_m)\tilde{m} + (\alpha_{11}F_e + \alpha_{12}G_e)\tilde{e} \\
\dot{\tilde{e}} &= (\alpha_{21}F_m + \alpha_{22}G_m)\tilde{m} + (\alpha_{21}F_e + \alpha_{22}G_e)\tilde{e}
\end{aligned} \tag{54}$$

For this system of differential equations, I need to have a stable equilibrium such that $\dot{\tilde{m}} = 0$ and $\dot{\tilde{e}} = 0$. For this, I define matrix \mathbf{A} as follows:

$$\mathbf{A} = \begin{pmatrix} \alpha_{11}F_m + \alpha_{12}G_m & \alpha_{11}F_e + \alpha_{12}G_e \\ \alpha_{21}F_m + \alpha_{22}G_m & \alpha_{21}F_e + \alpha_{22}G_e \end{pmatrix} \tag{55}$$

The stable equilibrium condition for the system is that the eigenvalues of \mathbf{A} must to be negative. Therefore the determinant of \mathbf{A} have to be positive and the trace of \mathbf{A} is supposed to be negative. Mathematically I can express this as:

$$\begin{aligned}
(\alpha_{11}\alpha_{22} - \alpha_{21}\alpha_{12})(F_m G_e - F_e G_m) &> 0 \\
\alpha_{11}F_m + \alpha_{12}G_m + \alpha_{21}F_m + \alpha_{22}G_m &< 0
\end{aligned} \tag{56}$$

If I want monetary policy to take care of domestic inflation and exchange rate policy of foreign inflation, then α_{12} and α_{21} are to be equal to zero. Therefore the following conditions apply:

$$\begin{aligned}
\alpha_{11}F_m + \alpha_{22}G_g &< 0 \\
\alpha_{11}\alpha_{22}(F_m G_g - F_g G_m) &> 0
\end{aligned} \tag{57}$$

I also assume that $F_m, G_e > 0$ therefore I need $\alpha_{11}, \alpha_{22} < 0$.

So the necessary condition for the system to reach equilibrium is:

$$\frac{G_e}{G_m} > \frac{F_e}{F_m} \quad (58)$$

Or equivalently:

$$\frac{\frac{\partial \pi_F}{\partial e}}{\frac{\partial \pi_F}{\partial m}} > \frac{\frac{\partial \pi_H}{\partial e}}{\frac{\partial \pi_H}{\partial m}} \quad (59)$$

Appendix B: Money supply and monetary rule

In order to derive a functional form of the money supply for the central bank, I will start by describing some identities for the high-powered money and the overall money measure, following closely McCallum (1989), but adapting this framework to add it in the DSGE structure.

Central bank only can changes high-powered money (H_t) to affect the amount of overall money in the economy. Broad money in the economy is described by the sum of currency and deposits ($M_t = C_t + D_t$), while high-powered money is described as currency and the commercial banks' reserves within the central bank ($H_t = C_t + TR_t$).⁶

Let $cr_t = C_t/D_t$ be the ratio of currency over deposits and $TR_t/D_t = rr + f(i_t)$ be the ratio of bank reserves over deposits for liquidity purpose.

I can decompose the bank reserves into required reserves (rr), as a percentage of the deposits and the excess reserves, ($f(i_t)$), which depends on the interest rate, with $f'(i_t) < 0$.

Therefore the ratio of broad money mass over high-powered money, also known as the money multiplier, is:

$$M_t = \frac{1+cr}{cr+rr+f(i_t)} H_t \quad (60)$$

The steady state ratio is:

$$\bar{M}_t = \frac{1+cr}{cr+rr+f(\bar{i})} \bar{H}_t \quad (61)$$

Log-linearizing this expression I get:

$$\frac{M_t}{\bar{M}_t} = \frac{cr+rr+f(\bar{i})}{cr+rr+f(i_t)} \frac{H_t}{\bar{H}_t} \Rightarrow \tilde{m} = \tilde{h} + \frac{cr+rr+f(\bar{i})}{cr+rr+f(i_t)} \quad (62)$$

And using a first order Taylor approximation, I have that:

$$\ln(cr+rr+f(i_t)) = \ln(cr+rr+f(\bar{i})) + (i_t - \bar{i}) \frac{1}{cr+rr+f(\bar{i})} f'(\bar{i}) \quad (63)$$

Therefore I get the following expression:

⁶ I assume that banks acquire deposits from households and invest them in government bonds.

$$\begin{aligned}\tilde{m} &= \tilde{h} - \frac{(i_t - \bar{i})}{cr + rr + f(\bar{i})} f'(\bar{i}) \\ \tilde{m} &= \tilde{h} - \frac{\tilde{i}_t}{cr + rr + f(\bar{i})} f'(\bar{i})\end{aligned}\tag{64}$$

In order to simplify parameterization, I will assume that $f(i)$ is a linear function of i ($f(\tilde{i}_t) = -\phi\tilde{i}_t$) and therefore I can express it as: $-\frac{\phi}{cr + rr + \phi\bar{i}}$. So I end up with this expression:

$$\tilde{m} = \tilde{h} + \underbrace{\frac{\phi}{cr + rr + \phi\bar{i}}}_{\xi} \tilde{i}\tag{65}$$

Using the last equation and combining it with (18) I get obtain money market equations. In order to simplify the calculations, I will write them as follows:

$$\begin{aligned}\tilde{m}_t &= \tilde{p}_t + \alpha_y \tilde{y}_t - \alpha_m \tilde{i}_t + \iota_t \\ \tilde{m}_t &= \tilde{h}_t + \xi \tilde{i}_t + n_t\end{aligned}\tag{66}$$

Where $\xi = \frac{\phi}{cr + rr + \phi\bar{i}}$ and n_t is a shock to the money supply.

Now I can equalize them and find out an equation that relates the interest rate with high powered money controlled by the central bank:

$$\tilde{i}_t = \frac{1}{\alpha_m + \xi} \left(\tilde{p}_t + \alpha_y \tilde{y}_t - \tilde{h}_t + \iota_t - n_t \right)\tag{67}$$

Since at this interest rate the money market is in equilibrium, now I need to find a money supply rule (rather than an interest rate rule) for the model.

To obtain suitable parameters for this law of motion, I will define Taylor-type rule for the interest in the typical way:

$$\tilde{i}_t = \rho_i \tilde{i}_{t-1} + \rho_y \tilde{y}_t + \rho_\pi \tilde{\pi}_t\tag{68}$$

Then I can rearrange the equilibrium money market expressions such as:

$$\tilde{h}_t = \tilde{p}_t + \alpha_y \tilde{y}_t - (\xi + \alpha_m) \tilde{i}_t + \psi_t\tag{69}$$

By plugging this expression, I obtain the money supply rule and disaggregating $\tilde{\pi}_t = \tilde{p}_t - \tilde{p}_{t-1}$, I get:

$$\begin{aligned}
\tilde{h}_t &= \tilde{p}_t - \underbrace{[(\alpha_m + \xi)\rho_\pi]}_{a_1} \tilde{\pi}_t + \underbrace{(\alpha_y - (\alpha_m + \xi)\rho_y)}_{a_2} \tilde{y}_t - \underbrace{(\alpha_m + \xi)\rho_i}_{a_3} \tilde{i}_{t-1} + \psi_t \\
&= \tilde{p}_t - a_1 \tilde{\pi}_t + a_2 \tilde{y}_t - a_3 \tilde{i}_{t-1} + \psi_t
\end{aligned} \tag{70}$$

Here I have a money supply rule consistent with the well-known Taylor interest rule. It is worth to note that as ρ_i must be greater than 1 in the Taylor rule, a_1 have to be negative. In the case of the output gap, it will be positive, but (if $\alpha_y = 1$ as I assume), will be less than 1.

Appendix C: Optimal choice of instruments

In this paper, I have introduced a money supply rule rather than an interest rate rule, without explaining the reason behind this. For justifying the choice of using the money supply rule, I use a Poole (1970) model for the optimal choice of instruments, modified by McCallum (1989). This model states that depending on the origin of the shocks, it is better to use either money supply or an interest rate rule.

Given the money demand and supply equations shown in (66), I will analyze the effect of using a money supply rule, (the central bank chooses \tilde{h}_t in order to cope with different shocks) and an interest rate rule. I start by adding money demand and supply shocks to both equations:

$$\begin{aligned}\tilde{m}_t &= \tilde{p}_t + \alpha_y \tilde{y}_t - \alpha_m \tilde{i}_t + \iota_t \\ \tilde{m}_t &= \tilde{h}_t + \xi \tilde{i}_t + n_t\end{aligned}\tag{71}$$

Where ι_t and n_t are money demand and supply shocks respectively.

I assume the Central Bank chooses to fix \tilde{h}_t or ι_t in advance shocks will happen. If in equilibrium the broad money target is m_t^T according the authority's target. Then I equalize both interest rates and solving for high-powered money to ensure reaching m_t^T . This gives us:

$$\tilde{h}_t^T = \frac{(\alpha_m + \xi)\tilde{m}_t^T - \xi E_{t-1}\tilde{p}_t - \alpha_y \xi E_{t-1}\tilde{y}_t}{\alpha_m}\tag{72}$$

Obviously realized and expected high-powered money will differ. Therefore I will get a broad money value that will vary from the one wanted due these shocks. To calculate this difference, I find \tilde{m}_t in given the \tilde{h}_t I just set:

$$\tilde{m}_t = m_t^T + \frac{\xi(\tilde{p}_t - E_{t-1}\tilde{p}_t) + \alpha_y \xi(\tilde{y}_t - E_{t-1}\tilde{y}_t) + \xi \iota_t + \alpha_m n_t}{\alpha_m + \xi}\tag{73}$$

To simplify our analysis, I will define a composite shock:

$$r_t = (\tilde{p}_t - E_{t-1}\tilde{p}_t) + \alpha_y (\tilde{y}_t - E_{t-1}\tilde{y}_t) + \iota_t\tag{74}$$

Replacing central bank's \tilde{h}_t set in advance, I get an expression for the difference of actual broad money and the target:

$$\tilde{m}_t - \tilde{m}_t^* = \frac{\xi n_t + \alpha_m r_t}{\xi + \alpha_m}\tag{75}$$

Finally, it is more useful to analyze the volatility of this difference in order to compare alternative procedures. Taking the variance of both sides, it results in:

$$\text{var}(\tilde{m}_t - \tilde{m}_t^*) = \frac{\xi^2 \sigma_n^2 + \alpha_m^2 \sigma_r^2}{(\xi + \alpha_m)^2} \quad (76)$$

Where I define σ_i^2 and σ_n^2 as variances of money demand and supply shocks. I can clearly see that either source of shock will affect the broad money difference.

Now I will compare this money supply rule to an interest rate rule, when the central bank sets the interest rate to make $\tilde{m} = \tilde{m}^*$. Then the difference between both in this case is:

$$\tilde{m}_t - \tilde{m}_t^* = r_t \quad (77)$$

So by taking the variance in both sides I get:

$$\text{var}(\tilde{m}_t - \tilde{m}_t^*) = \sigma_r^2 \quad (78)$$

So the difference only depends on the money demand shock. By comparing both results, I can see that when the shock comes from the money demand, it is better to use is the money supply rule since this absorbs part of the shock. In the case when there are bigger money supply shocks, the best instrument is an interest rate rule, since it avoids completely the shock. In fact, if $\sigma_r^2 = \sigma_n^2$ a high-powered rule will be useful due denominator will be higher than the numerator.⁷

⁷ I exclude some caveats about the signaling effect of interest rate in this case.